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Original Article

FUZZY TRANSSHIPMENT PROBLEM VIA METHOD OF MAGNITUDE

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ABSTRACT

This paper deals with transshipment problem in fuzzy environment where before reaching its actual destination the available commodity frequently moves from one source to another source. The Trapezoidal Fuzzy numbers are deffuzified using method of Magnitude Technique. The efficient solutions for the Fuzzy transshipment problem is determined with various methods like North west corner method(NWCM), Matrix Minima method (MMM) and Vogel's approximation method (VAM).The solutions obtained by these methods are compared to identify the best optimal solution.

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KEYWORDS: Fuzzy Transshipment Problem, Trapezoidal Fuzzy Numbers, Method of Magnitude, Vogel's Approximation Method, MODI Method.

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INTRODUCTION

Normally for a transportation problem shipments will be made only between source-sink pairs. The situations where a commodity is shipped from sources to destinations and their capacities are $a_1, a_2, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ respectively are handled with the Transportation problem. The objective is to minimize the total shipped cost while satisfying both the supply limits and the demand requirements and to determine the amount shipped from each source to each destination. The units of a product can be transshipped from a source to sink through the exist points. Transshipment problems are problems with these additional features. Any given transshipment problem can be converted easily into an equivalent transportation problem .In case of a transshipment problem all the source and destinations can function in any direction.. The transportation cost goes higher in the usually, in the absence of transshipment. For reducing the transportation cost the transshipment problem can be used as an alternate way. Orden (1956) [1] has extended transportation problem to include the case when transshipment is also allowed. The optimal solution for a transshipment problem can be found by solving a transportation problem by defining a supply point which will be used as a one way for sending the commodities alone and cannot be utilized for receiving the commodities from any other point. By the same procedure the demand point will also act as a one way point which will be utilized for receiving the commodities and cannot be used for sending the commodities. Shore (1970) [2] have developed the transportation problem and the Vogel's approximation method. The transportation problem with the mixed constraints is developed by Bridgen (1974) [3]. A transshipment point is a point that can both receive goods from other points and send goods to other points. Aneja (1979) [4] had presented about bi - criteria transportation problem. The time minimizing transshipment

problem and the optimal routes of Transportation from origin to destination with Transshipment was developed by Garg and Prakash (1985) [5]. Gupta (1993) [6] had discussed about the fractional transportation problems with mixed constraints. Ozdemir (2006) [7] explained about the multi-location transshipment problem with capacitated production and lost sales.

Zadeh (1965) [8] was the first to investigate the concept of Fuzzy set .The arithmetic's operations with these fuzzy numbers have been introduced by Bellman and Zadeh (1970) [9] and Kaufmann (1976) [10]. Nagoor Gani et al (2004) [11] have developed the solution of Transportation Problem by using Fuzzy Number. Adlakha (2006) [12] have found the transportation problems with mixed constraints. A two stage cost minimizing fuzzy transportation problem in which supplies and demands are trapezoidal fuzzy number have been solved by Nagoor Gani et al(2006) [13]. Nagoor Gani et al (2011)[14] proposed an algorithm for modifying and solving the mixed constraint fuzzy transshipment problem into a standard fuzzy transshipment problem.

Abirami et al (2012) [15] had proposed an algorithm to yield an optimum solution of the fuzzy transshipment problem. The main advantage of the proposed method is solved manually, without using artificial variables which is required by the simplex method also it reduces number of iterations. Nagoor Gani et al (2014) [16] presented an improved version of Vogel's Approximation Method (IVAM) to find the efficient initial solution for the large scale transshipment problems. Performance of IVAM over VAM is discussed. The Intuitionstic fuzzy transshipment problem was converted into an transportation problem and the optimal solution was obtained by Nagoor Gani et al (2014) [17].

Large scale fuzzy transshipment problem was discussed by Gayatri et al (2015)[18] and a new method called Max-Min Method was implemented. Mohanapriya (2016)[19] determined the efficient solutions for the large scale Fuzzy transshipment problem by using Vogel's Approximation Method (VAM).

The paper is designed as follows; basic definitions, fuzzy transshipment problem and method of magnitude have been discussed in section 2.In section 3 Algorithms for various Methods are discussed. A numerical example is explained with the help of section 2 and section3.In section 4 Comparison with the various methods and conclusion is given.

PRELIMINARIES

Fuzzy Set

A fuzzy set **A** in **X** is characterized by a membership function $f_A : X \rightarrow [0,1]$, $x \in A$

Fuzzy Number

A fuzzy set **A** on R must possess at least the following three properties to qualify as a fuzzy number.

- **A** must be a normal fuzzy set.
- $\mathbf{A}_\alpha$ must be a closed interval for every $\alpha \in [0,1]$
- The support **A**, $^{0+}\mathbf{A}$ must be bounded.

Trapezoidal and Triangular Fuzzy numbers

If the membership function $f_{\tilde{A}}(x)$ is piecewise linear, Then $\tilde{A}$ is said to be a trapezoidal fuzzy number. The membership function of a trapezoidal fuzzy number is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{w(x-a)}{b-a} \, if \ a \le x \le b \\ w \ if \ b \le x \le c \\ \dfrac{w(x-d)}{(c-d)} \, if \ c \le x \le d \\ 0 \quad otherwise \end{cases}$$

If $w = 1$, then $\tilde{A} = (a, b, c, d; 1)$ is a normalized trapezoidal fuzzy number and $\tilde{A}$ is a generalized or non normal trapezoidal fuzzy number if $0 < w < 1$.The image of $\tilde{A} = (a, b, c, d; w)$ is given by $-\tilde{A} = (-d, -c, -b, -a; w)$.

In particular case if $b = c$, the trapezoidal fuzzy number reduces to a triangular fuzzy number given by $\tilde{A} = (a, b, d; w)$. The value of "b" corresponds with the mode or core and $[a, d]$ with the support. If $w = 1$, then $\tilde{A} = (a, b, d)$ is a normalized triangular fuzzy number $\tilde{A}$ is a generalized or non normal triangular fuzzy number if $0 < w < 1$.

Properties of Trapezoidal Fuzzy Number

Let $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then the fuzzy numbers addition and fuzzy numbers subtraction are defined as follows

- Fuzzy numbers addition of $\tilde{A}$ and $\tilde{B}$ is denoted by $\tilde{A} \oplus \tilde{B}$ and is given by
- $\tilde{A} \oplus \tilde{B} = (a_1, b_1, c_1, d_1) \oplus (a_2, b_2, c_2, d_2) = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
- Fuzzy numbers subtraction of $\tilde{A}$ and $\tilde{B}$ is denoted $\tilde{A} \ominus \tilde{B}$ and is given by
- $\tilde{A} \ominus \tilde{B} = (a_1, b_1, c_1, d_1) \ominus (a_2, b_2, c_2, d_2) = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$

New Approach for Ranking of Trapezoidal Fuzzy Numbers

Method of Magnitude

For an arbitrary trapezoidal fuzzy number $u = (x_0, y_0, \sigma, \beta)$, with parametric form

$u = (\overline{u}(r), \underline{u}(r))$, we define the magnitude of the trapezoidal fuzzy number u as

$$\text{Mag}(u) = \frac{1}{2}\left(\int_0^1 \left(\overline{u}(r) + \underline{u}(r) + x_0 + y_0\right) f(r) dr\right),$$

where the function f(r) is a non-negative and increasing function on [0, 1] with

$f(0) = 0$, $f(1) = 1$ and $\int_0^1 f(r) dr = 1$.

For example, we can use $f(r) = r$, to rank the fuzzy numbers.

For any two trapezoidal fuzzy numbers $u, v \in E$, The ranking of u and v by magnitude on E is given by,

- Mag(u) > Mag(v) if and only if u $\succ$ v,
- Mag(u) < Mag(v) if and only if u$\prec$v,
- Mag (u) = Mag (v) if and only if u $\approx$ v.

Then we formulate the order $\succcurlyeq$and $\preccurlyeq$ as u $\succcurlyeq$ v if and only if u $\succ$ v or u $\approx$ v, u $\preccurlyeq$ v

If and only if u $\prec$ v or u $\approx$ v.

Method of Promoter Operator

Let u = (a, b, c, d) be a non-normal trapezoidal fuzzy numbers with *r*-cut representation

u = ($\overline{u}$(r), u(r)), consequently we have

Mag (u) = $\frac{(3w^2+2)(b+c)}{12w}$, $\frac{(3w-2)(a+d)}{12w}$

For normal trapezoidal fuzzy numbers the above formula can be reduced to

Mag (u) = $\frac{5}{12}(b+c)+\frac{1}{12}(a+d)$

Formulation of the Fuzzy Transshipment Problem

The fuzzy transportation problem assumes that direct routes exist from each source to each destination. In Some circumstance there may arise a critical situations where before reaching the destinations the commodities can be shipped from one source to another destinations. This is called a fuzzy transshipment problem. A transportation problem with m source and n destination give rise to a Transshipment Problems with (m+n) source and (m+n) destinations. The basic feasible solution to such a problem will have [(m + n) + (m + n) -1] or [2m + 2n -1] basic variables and if we omit the variables appearing in the (m + n) diagonal cells, we are left with (m + n – 1) basic variables.

The fuzzy transshipment problem may be mathematically written as:

Minimize $\tilde{z} = \sum_{i=1}^{m+n}\sum_{j=1,j\neq i}^{m+n}\tilde{c}_{ij}\tilde{x}_{ij}$

Subject to $\sum_{j=1,j\neq i}^{m+n}\tilde{x}_{ij}-\sum_{j=1,j\neq i}^{m+n}\tilde{x}_{ji}=\tilde{a}_i$, i=1,2,3…m

$\sum_{i=1,i\neq j}^{m+n}\tilde{x}_{ij}-\sum_{i=1,i\neq j}^{m+n}\tilde{x}_{ji}=\tilde{b}_j$, j= m+1,m+2,…m+n

Where $\tilde{x}_{ij}\geq 0$, i, j=1,2, 3,…..,m+n,j $\neq$i

Where $\sum_{i=1}^{m}\tilde{a}_i=\sum_{j=1}^{m}\tilde{b}_j$ then the problem is balanced otherwise unbalanced.

The l reduced transportation form is given by

Minimize $\tilde{z} = \sum_{i=1}^{m+n}\sum_{j=1,j\neq i}^{m+n}\tilde{c}_{ij}\tilde{x}_{ij}$

Subject to $\sum_{i-1}^{m+n} \breve{x}_{ij} = \breve{a}_i + T$, i =1, 2, 3,…,m

$\sum_{j=1}^{m+n} \breve{x}_{ij}$ =, i=m+1,m+2,m+3,….m+n

$\sum_{i=1}^{m+n} \breve{x}_{ij}$ =, j =1, 2, 3,…, m

$\sum_{i=1}^{m+n} \breve{x}_{ij} = \breve{b}_j + T$, j=m+1,m+2,m+3,….m+n

Where $\breve{x}_{ij} \geq 0$, i ,j=1,2,3,…..,m+n,j $\neq$i.

Since a large amount of commodities can be transshipped at each points T can be considered as a buffer stock at each origin and destination. T should balance the amount produced or received and it is given as

T= $\sum_{i=1}^{m} \breve{a}_i$ or $\sum_{j=1}^{m} \breve{b}_j$

ALGORITHMS

The transshipment table in fuzzy environment is similar to fuzzy transportation table [19]. The fuzzy transshipment problem can be solved by the three given algorithms namely, North West Corner Method, Least Cost Method and Vogel's Approximation Method. Before applying these methods the Trapezoidal fuzzy numbers are converted into crisp numbers by using the method of magnitude ranking technique.

- The transshipment problem is modified into a standard transportation problem.

Convert the Trapezoidal fuzzy numbers to crisp numbers by method of magnitude ranking technique for each cell of transportation table.

- Initial basic feasible solution can be determined by using any one of the three given methods explained in 4.1, 4.2 and 4.3.
- The Final Optimum solution can be obtained by using MODI method.

North-West Corner Method (NWCM)

In this method the basic variables are selected from the North – West corner (i.e., top left corner).

Steps

- Choose the North West corner of the transportation table and allocate as many units as possible equal to the minimum supply and demand. i.e., min (s1, d1).
- The supply and demand numbers in the respective rows and column allocations are adjusted.
- Move down to the first cell in the second row if the supply for the first row is exhausted.
- Move down horizontally to the next cell in the second column If the supply for the first row is exhausted.
- Make the next allocation in cell either in the next row or column if for any cell supply equals demand.
- Continue the procedure until the total available quantity is fully allocated to the cells as required.

Matrix Minima Method (MMM)

In this method the basic variables are selected according to the unit cost of transportation

Steps

- Examine the box having minimum unit transportation cost (cij).
- Select the row and the column corresponding to the lower numbered row if there are two or more minimum costs. If they appear in the same row then select the lower numbered row.
- By considering the capacity and requirement constraints identify the lowered numbered column.
- Delete the column if the demand is satisfied.
- Delete the row if the supply is exhausted.
- The steps 1-5 are repeated until all the restrictions are satisfied.

Vogel's Approximation Method (VAM)

This method is widely used for solving the transportation problem.

- Select the boxes having the minimum and the next minimum transportation cost in each row/column and write the penalty along the side of the table against the corresponding row/column.
- Find out the maximum penalty along the side /below the table and make maximum allotment to the box having minimum cost of transportation in that row/column.
- Select the top most rows and the extreme left column if the penalties corresponding to two or more rows or columns are equal.
- Repeat this process until all the conditions are satisfied.

Modified Distribution Method (MODI)

Steps

- Obtain the initial basic feasible solution using any one of the three given methods namely **NWCM, MMM** and **VAM.**
- Find the values of dual variables, u_i and v_j, using $u_i + v_j = c_{ij}$
- Compute the opportunity cost using dij= cij – (ui + vj) from unoccupied cell.
- Identify the sign of each opportunity cost (dij). The given solution is the optimum solution if the opportunity costs of all the unoccupied cells are either positive or zero. At the same time the given solution is not an optimum solution if one or more unoccupied cell has negative opportunity cost,
- For Proceeding with the next solution choose the unoccupied cell with the smallest negative opportunity cost.
- Following the previous step draw a closed path or loop for the unoccupied cell. Take in account that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.

- Use alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.
- Calculate the maximum number of units that should be shipped to this unoccupied cell. Lowest value with a negative position on the closed path is identified as the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. Hence an unoccupied cell becomes an occupied cell.
- The whole procedure is repeated until an optimum solution is reached.

Numerical Example

Consider the transshipment problem with two origins and two destinations.

Table 1

	$\breve{D}_1$	$\breve{D}_2$	
$\breve{O}_1$	**(2,3,6,6)**	**(1,4,7,7)**	**(1,2,18,18)**
$\breve{O}_2$	**(3,4,8,8)**	**(5,6,9,9)**	**(6,8,10,10)**
	(4,6,10,10)	**(3,4,18,18)**	**(7,10,28,28)**

Table 2

	$\breve{D}_1$	$\breve{D}_2$
$\breve{D}_1$	**(0,0,0,0)**	**(2,3,6,6)**
$\breve{D}_2$	**(5,6,9,9)**	**(0,0,0,0)**
	$\breve{O}_1$	$\breve{O}_2$
$\breve{O}_1$	**(0,0,0,0)**	**(2,5,18,18)**
$\breve{O}_2$	**(1,1,10,10)**	**(0,0,0,0)**

Fuzzy transshipment problem table

Table 3

	$\breve{O}_1$	$\breve{O}_2$	$\breve{D}_1$	$\breve{D}_2$	
$\breve{O}_1$	(0,0,0,0)	(2,5,18,18)	(2,3,6.6)	(1,4,7,7)	(8,12,46,46)
$\breve{O}_2$	(1,1,10,10)	(0,0,0,0)	(3,4,8,8)	(5,6,9,9)	(13,18,38,38)
$\breve{D}_1$	(4,6,10,10)	(3,4,18,18)	(0,0,0,0)	(2,3,6,6)	(7,10,28,28)
$\breve{D}_2$	(1,4,7,7)	(6,6,21,21)	(5,6,9,9)	(0,0,0,0)	(7,10,28,28)
	(7,10,28,28)	(7,10,28,28)	(11,16,38,38)	(10,14,46,46)	

By using the method of magnitude ranking technique the trapezoidal fuzzy numbers are converted into crisp numbers as given below

Mag (u) = $\frac{5}{12}(b+c)+\frac{1}{12}(a+d)$,(2, 5, 18, 18) = $\frac{5}{12}(5+18)+\frac{1}{12}(2+18)$ = 11.1

Table 4

$\tilde{O}_1$	0	11.1	4.4	5.2	28.6
$\tilde{O}_2$	5.5	0	5.9	7.4	27.5
$\tilde{D}_1$	7.1	10.5	0	4.4	18.75
$\tilde{D}_2$	5.25	13.5	7.4	0	18.75
	18.75	18.5	26.5	29.6	

North-West Corner Method (NWCM)

by using the algorithm 4.1 .we get the optimum table given below

Table 5

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$
$\tilde{O}_1$	0 (18.75)	11.1 (9.85)	4.4	5.2
$\tilde{O}_2$	5.5	0 (8.9)	5.9 (18.6)	7.4
$\tilde{D}_1$	7.1	10.5	0 (7.9)	4.4 (10.85)
$\tilde{D}_2$	5.25	13.5	7.4	0

(11.1 x 9.85)+ (5.9 x 18.6) + (4.4 x 10.85) = 266.81

Matrix Minima Method (MMM)

by using the algorithm 4.2 .we get the optimum table given below

Table 6

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$
$\tilde{O}_1$	0 (18.75)	11.1	4.4 (7.75)	5.2 (2.1)
$\tilde{O}_2$	5.5	0 (18.75)	5.9	7.4
$\tilde{D}_1$	7.1	10.5	0 (18.75)	4.4
$\tilde{D}_2$	5.25	13.5	7.4 (8.75)	0 (18.75)

(4.4 x 7.75)+ (5.2 x 2.1) + (7.4 x 8.75) = 109.77

By using Vogel's Approximation Method (VAM)

by using the algorithm 4.3 .we get the optimum table given below

Table 7

	$\tilde{O}_1$	$\tilde{O}_2$	$\tilde{D}_1$	$\tilde{D}_2$
$\tilde{O}_1$	0 (18.75)	11.1	4.4	5.2 (9.85)
$\tilde{O}_2$	5.5	0 (18.75)	5.9 (7.75)	7.4
$\tilde{D}_1$	7.1	10.5	0 (18.75)	4.4
$\tilde{D}_2$	5.25	13.5	7.4 (1)	0

(5.2 x 9.85)+ (7.4 x 1) + (5.9 x 7.75) = 102.87

The optimum solution is obtained using MODI method is **102.87**

The same fuzzy transshipment problem has been converted into a crisp general transportation problem using Yager's ranking index by Mohana priya et al, and the optimum solution by VAM and MODI method is given as **224.01.**

By our method of magnitude ranking method the optimal solution by the above three methods are given below.

NWCM	LCM	VAM
266.81	109.77	102.87

CONCLUSIONS

In this paper, the trapezoidal fuzzy transshipment problem has been converted into a crisp general transportation problem using Method of Magnitude. The cost at the origins and destinations are all symmetric trapezoidal fuzzy numbers and the solution to the problem is given by three different methods. Out of these methods the solution by VAM method is optimum.

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